ILLINOIS

Probability

Learning Objectives

1. Know the definitions presented
2. Understand the types of randomized algorithms

Fundamentals of Probability

Imagine you roll a pair of six-sided dice.

The **sample space** Ω is the set of all possible outcomes.

An **event** $E \subseteq \Omega$ is any subset.

Fundamentals of Probability

Imagine you roll a pair of six-sided dice.

A **random variable** is a function from events to numeric values.

A **distribution** of a random variable ***X*** :

Fundamentals of Probability

Imagine you roll a pair of six-sided dice. What is the expected value?

The **expectation** of a (discrete) random variable is:

$$E[X] = \sum_{x \in \Omega} Pr\{X = x\} \cdot x$$

Fundamentals of Probability

Imagine you roll a pair of six-sided dice. What is the expected value?

$$E[X + Y] = ?$$

Fundamentals of Probability

Imagine you roll a pair of six-sided dice. What is the expected value?

Linearity of Expectation: For any two random variables X and Y,

$$E[X+Y] = E[X] + E[Y]$$

Fundamentals of Probability

Linearity of Expectation: For any two random variables X and Y,

Randomization in Algorithms

1. Assume input data is random to estimate average-case performance
   - BST
   - Experimenting

2. Use randomness inside algorithm and estimate expected running time
   - QuickSort, HashTable

3. Use randomness inside algorithm to approximate solution in fixed time

Quick Primes with Fermat's Primality Test

If p is prime and a is not divisible by p, then $a^{p-1} \equiv 1 (mod\, p)$

But… ***sometimes*** if n is composite and $a^{n-1} \equiv 1 (mod\, n)$